OBJECTIVE MATHEMATICS

Volume 1

Descriptive Test Series

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CHAPTER-8 : BINOMIAL THEOREM

UNIT TEST-1

1. In the expansion of $(x+a)^n$ if the sum of odd terms be P and sum of even terms be Q, then

(a) $P^2 - Q^2 = (x^2 - a^2)^n$

(b) $4\,PQ = (x+a)^{2n} - (x-a)^{2n}$

2. Let $\left(\dfrac{2x^2+x+2}{x}\right)^n = \sum_{r=m}^{r=t} a_r x^r$,

then answer the following.

(a) The values of m and t are ...

(b) The value of $\sum_{r=m}^{r=t} a_r = \ldots$

(c) The value of $a_t = \ldots$

(d) The value of $a_m = \ldots$

(e) If $a_p = a_q$, then $p + q =$

$$E = \left[1+2\left(x+\frac{1}{x}\right)\right]^n = \sum_{r=m}^{r=t} a_r x^r,$$

where $r = (m, m + 1,\ldots, t)$

$$E = C_0 + C_{1.2}\left(x+\frac{1}{x}\right) + C_2\left[2\left(x+\frac{1}{x}\right)\right]^2 +$$

$$+C_n\left[2\left(x+\frac{1}{x}\right)\right]^n = \sum_{r=m}^{t} a_r x^r$$

3. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, then the sum of the products of the C^s_i taken two at a time represented by $\sum\sum C_i C_j$ is equal to $0 \le i \le j \le n$.

$$2^{2n-1} - \frac{(2n)!}{2.(n!)^2}.$$

4. $C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n (C_n)^2 = 0$

or $(-1)^{n/2} \dfrac{n!}{(n/2)!(n/2)!}$

according as n is odd or even.

5. If the maximum value of the term independent of $\left(\dfrac{1}{1-x}\right)\left(\dfrac{1}{3-x}\right)$ is :

6. If C_r stands for ${}^nC_r$, find the sum of the series $\dfrac{2(n/2)!(n/2)!}{n!}\left[C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)C_n^2\right.$

where n is an even positive integer, is :

7. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \dfrac{(1-x)^{\frac{1}{10}}}{t}\right), x \ge 0$ is k, then k is equal to ....

Hints and Solutions

1. (True) $P + Q = (x + a)^n$

$\therefore$ $(P - Q) = (x - a)^n$

$\therefore$ $P^2 - Q^2 = (x^2 - a^2)^n$

$4PQ = (P + Q)^2 - (P - Q)^2$

$= (x + a)^{2n} - (x - a)^{2n}$

2. (a) Clearly $m = -n$, $t = n$ as the highest power of x will be n and lowest will be $-n$.

(b) $\sum a_r =$ Sum of binomial coefficients is obtained by putting $x = 1$ and hence

$$\sum a_r = 5^n.$$

(c) $a_t = a_n =$ coefficient of $x^n = {}^nC_n\, 2^n = 2^n$.

(d) $a_m = a^{-n} =$ coefficient of $x^{-n} = {}^nC_n\, 2^n = 2^n$

(e) $a_p = a_q$, then if $p = \lambda$ then $q = -\lambda$

$\therefore p + q = 0$ by (d).

3. True

$$\sum\sum C_i C_j = \frac{1}{2}\left[\left(\sum C_i\right)^2 - \sum C_i^2\right]$$

$$0 \le i < j \le n$$

$$= \frac{1}{2}\left[(2^n)^2 - \frac{(2n)!}{(n!)^2}\right],$$

$$= 2^{2n-1} - \frac{(2n)!}{2(n!)^2}$$

$$C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = \frac{(2n)!}{n!n!}$$

$(1+x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \ldots$

$$\left(1+\frac{1}{x}\right)^n = C_0 + C_1\frac{1}{x} + C_2\frac{1}{x^2} + C_3\frac{1}{x^3} + \ldots$$

Multiplying, we observe that

$C_0^2 + C_1^2 + C_2^2 + \ldots$ is the term independent of x in

$$(1+x)^n\frac{(1+x)^n}{x^n} \text{ or in } \frac{(1+x)^{2n}}{x^n}$$

or the coefficient of x^n $(1+x)^{2n}$ which will be

$$^{2n}C_n = \frac{(2n)!}{n!n!}$$

4. (True)

$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$

$(x - 1)^n = C_0 x^n - C_1 x^{n-1} + C_2 x^{n-2} + \ldots + (-1)^n C_n$

Multiplying both sides, $(-1)^n (1 - x^2)^n = (\ )(\ )$

Now $C_0^2 - C_1^2 + C_2^2 - \ldots$ is the coefficient of x^n in the product in R. H. S.

Hence it is the coeff. of x^n in $(-1)^n (1 - x^2)^n$ or coeff. of $(x^2)^{n/2}$ in $(1 - x^2)^n$ which will appear in

$T_{\frac{n}{2}+1}$

$\therefore\ (-1)^n\, {}^nC_{n/2}\, (-1)^{n/2} (x^2)^{n/2}$

Above is possible only when $n/2$ is an integer *i.e., n* is even and in case n is odd, then the terms x^n will not occur and hence answer is zero. Also when n is even, then $(-1)^n = 1$

$\therefore\ (-1)^{n/2}.\dfrac{n!}{\frac{n}{2}!\frac{n}{2}!}$ is the required answer.

5. $$\frac{1}{(1+x)(3-x)} = (1-x)^{-1} = (3-x)^{-1}$$

$$= 3^{-1}(1-x)^{-1}\left(1-\frac{x}{3}\right)^{-1}$$

$$= \frac{1}{3}[1+x+x^2+\ldots x^n]\left[1+\frac{x^2}{3}+\frac{x^2}{3^2}+\ldots+\frac{x^{n-1}}{3^{n-1}}+\frac{x^n}{3^n}\right]$$

Coefficient of $x^n = \dfrac{1}{3^{n+1}} + \dfrac{1}{3^n} + \dfrac{1}{3^{n-1}} + \ldots(n+1)$ terms

$$= \frac{1}{3^{n+1}}\cdot\frac{[3^{n+1}-1]}{3-1}. = \frac{3^{n+1}-1}{2\cdot 3^{n+1}}$$

6. We have $C_0^2 - 2C_1^2 + 3C_2^2 - \ldots + (-1)^n(n+1)\, C_n^2$

$= [C_0^2 - C_1^2 + C_2^2 - \ldots + (-1)^n C_n^2] - [C_1^2 - C_2^2 + 3C_3^2 \ldots + (-1)^n n\, .\, C_n^2$

$= (-1)^{n/2}\, .\, {}^nC_{n/2} - (-1)^{n/2-1}$

$$\frac{1}{2}n.{}^nC_{n/2} = (-1)^{n/2}\left[1+\frac{n}{2}\right].nC_{n/2}$$

Therefore the value of given expression

$$= \frac{2.\frac{n}{2}!\frac{n}{2}!}{n}\left[(-1)^{n/2}.\left(1+\frac{n}{2}\right)\frac{n}{\frac{n}{2}!\frac{n}{2}!}\right] = (-1)^{n/2}(n+2)$$

7. (6006)

General Term $= 15C_r\left(t^2x^{\frac{1}{5}}\right)^{15-r}\left(\dfrac{(1-x)^{\frac{1}{10}}}{t}\right)^r$

For term independent on t

$2(15 - r) - r = 0 \Rightarrow r = 10 \therefore\ T_{11} = {}^{15}C_{10}\, x\,(1 - x)$

Maximum value of $x\,(1 - x)$ occur at

$$x = \frac{1}{2}$$

i.e., $(x(1-x))_{\max} = \dfrac{1}{4} \Rightarrow k = {}^{15}C_{10} \times \dfrac{1}{4}$

$\Rightarrow\quad 8\,k = 2({}^{15}C_{10}) = 6006$